Mass Conservation and Positivity Preservation with Ensemble-type Kalman Filter Algorithms

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Maintaining conservative physical laws numerically has long been recognized as being important in the development of numerical weather prediction (NWP) models. In the broader context of data assimilation, concerted efforts to maintain conservation laws numerically and to understand the significance of doing so have begun only recently.

In order to enforce physically based conservation laws of total mass and positivity in the ensemble Kalman filter, we incorporate constraints to ensure that the filter ensemble members and the ensemble mean conserve mass and remain nonnegative through measurement updates. We show that the analysis steps of ensemble transform Kalman filter (ETKF) algorithm and ensemble Kalman filter algorithm (EnKF) can conserve the mass integral, but do not preserve positivity. Further, if localization is applied or if negative values are simply set to zero, then the total mass is not conserved either. In order to ensure mass conservation, a projection matrix that corrects for localization effects is constructed. In order to maintain both mass conservation and positivity preservation through the analysis step, we construct a data assimilation algorithms based on quadratic programming and ensemble Kalman filtering.

Mass and positivity are both preserved by formulating the filter update as a set of quadratic programming problems that incorporate constraints. Some simple numerical experiments indicate that this approach can have a significant positive impact on the posterior ensemble distribution, giving results that are more physically plausible both for individual ensemble members and for the ensemble mean. The results show clear improvements in both analyses and forecasts, particularly in the presence of localized features. Behavior of the algorithm is also tested in presence of model error.